Art of Problem Solving Online School

Are You Ready For

Introduction to Programming with Python

This diagnostic test consists of two parts, **Fundamentals** and **Problem Solving**. If your student can solve nearly all of the **Fundamentals** problems and at least half of the **Problem Solving** problems, **without using a calculator**, then the student is ready for the Art of Problem Solving Online Class **Introduction to Programming with Python**.

Fundamentals

1. **Arithmetic with negative numbers**. Compute:

   (a) $-2 + 6 + (-5)$
   (b) $19 - (-13)$
   (c) $(-5) \times (-8)$
   (d) $48 \div (-6)$

2. **Number theory basics**.

   (a) Find the least common multiple of 54 and 24.
   (b) Find the greatest common divisor of 288 and 684.
   (c) Find the remainder when $237 + (128 \cdot 386)$ is divided by 9.

3. **Working with variables**.

   (a) Expand the product $9(3x + 7)$.
   (b) Simplify $7a - 5b + 3(6a + b)$.
   (c) Simplify $3(5 - 2r) - 2(-3r + 1)$.
   (d) Expand the product $(4n + 1)(4n + 3)$.

4. **Linear equations**. Solve each of the following equations:

   (a) $3r - 4 = 16 - 7r$
   (b) $\dfrac{2x - 3}{5} = \dfrac{4 - 3x}{7}$
   (c) $2 - \dfrac{t}{4} = 3\left(5 - \dfrac{t}{6}\right)$

5. **Word problems**.

   (a) Every large pack of gum has 8 pieces and every small pack of gum has 3 pieces. Mary has 7 small packs and 4 large packs. How many pieces of gum does Mary have altogether?
   (b) Jason has 17 pencils and Shannon has 43 pencils. How many pencils must Shannon give Jason in order for them to have the same number of pencils?
   (c) Ravi's age is double Ranu's age. The product of their ages in years is 72. How old is Ranu?

(d) Billy brought a large box of chocolates to school. In his first class, he gave half of the chocolates in the box to his teacher. In his second class, he gave half of his remaining chocolates to his best friend. At lunch, he ate half of his remaining chocolates. If he had 3 pieces of chocolate left at the end of lunch, then how many pieces were in the box when Billy arrived at school?

Problem Solving

6. What is the value of the sum $5 + 10 + 15 + \cdots + 95 + 100$?

7. How many multiples of 7 are between 83 and 229?

8. Douglas writes down his favorite number, which is a two-digit positive integer. He then turns the number into a three-digit number by writing a 7 at the end of his favorite number. This new number is 385 more than Douglas's favorite number. What is Douglas's favorite number?

9. At Annville Junior High School, 30% of the students in the Math Club are in the Science Club, and 80% of the students in the Science Club are in the Math Club. There are 15 students in the Science Club. How many students are in the Math Club?

10. Anna writes the first 100 positive integers. She then circles the even integers in her list with a green pen. Then, Bob circles the multiples of five with a blue pen. How many numbers are circled exactly once?

11. Describe all values of x that satisfy $7 - 3x < x - 1 < 2x + 9$.

Answers:

1. (a) -1 (b) 32 (c) 40 (d) -8
2. (a) 216 (b) 36 (c) 1
3. (a) $27x + 63$ (b) $25a - 2b$ (c) 13 (d) $16n^2 + 16n + 3$
4. (a) $r = 2$ (b) $x = \frac{41}{29}$ (c) $t = 52$
5. (a) 53 (b) 13 (c) 6 (d) 24
6. 1050
7. 21
8. 42
9. 40
10. 50
11. $x > 2$